SAINTS COLLEGE
TRIVANDRUM

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**F – 4051**

**Reg. No. :** .....

**Name :** .....

**Third Semester B.Sc. Degree Examination, January 2019**
**First Degree Programme Under CBCSS**
**Complementary Course for Physics**
**MM – 1331.1 : MATHEMATICS – III**
**Differential Equations, Theory of Equations and Theory of Matrices**
**(2014 Admission Onwards)**

Time : 3 Hours

Max. Marks : 80

**All** the **10** questions are **compulsory**. **Each** question carries **1** mark.

1. Write the 6th degree polynomial equation having roots $\sqrt{2}$, $i\sqrt{2}$ and $i\sqrt{3}$.
2. Solve the equation $x^5 - 16x = 0$.
3. Find the smallest interval with integer end points containing the positive root of the equation $x^2 - 3 = 0$.
4. If $f(-1) = 1$ and $f(1) = -1$, is there a root for the equation $f(x) = 0$ in $(-1, 1)$ ?
5. Solve the initial value problem $\frac{dy}{dt} - y = 0$; $y(0) = \pi$.
6. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$.
7. Find the particular integral of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{5x}$.
8. Find the rank of the matrix $\begin{bmatrix} 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$.
9. Find $A^3$ if A is the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.
10. Find the eigenvalues of the matrix $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ -5 & 0 & 0 \end{bmatrix}$.

P.T.O.

Answer **any 8** questions from among the questions from **11** to **22**. **Each** question carries **2** marks.

11. Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in arithmetic progression.

12. Solve the equation $x^3 + x^2 + x + 1 = 0$.

13. Transform the equation $x^3 + 3x^2 + 5x + 1 = 0$ into an equation lacking the second term.

14. If $\alpha, \beta, \gamma$ are the roots of the polynomial equation $f(x) = 0$, prove that $f'(x) = \left(\frac{1}{x-\alpha} + \frac{1}{x-\beta} + \frac{1}{x-\gamma}\right) f(x)$.

15. Find the general solution of $e^x y' - y^2 = 0$.

16. Solve $y' + y - x = 0$.

17. Solve $xy' + y = x^4 y^4$.

18. Solve $y'' + 2y' + 5y = 0$.

19. Solve that inverse of an orthogonal matrix is orthogonal.

20. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, show that $A^2 - 4A + 3I = 0$.

21. Find the eigenvalues of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$.

22. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 4 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix}$.

Answer **any 6** questions from among the questions from **23** to **31**. **Each** question carries **4** marks.

23. Solve $x^4 + x^3 + 33x^2 + 61x - 14 = 0$, given $2 + \sqrt{3}$ is a root.

24. Using Newton-Raphson method, find a root of the equation $x^3 + x - 1 = 0$ correct to 6 decimal places ($x_0 = 1$).

25. Using Descarte's rule of signs, show that the equation $12x^7 - x^4 + 10x^3 - 28 = 0$ has at least four imaginary roots.

26. Find the orthogonal trajectories of the family of parabolas $y = cx^2$.

27. Solve $(y - 2x^3)dx - x(1 - xy)dy = 0$.

28. Solve $y'' - 13y + 12y = e^{-2x}$.

29. Solve $x^2y' + 2xy - \sin x = 0$.

30. Test for consistency and solve $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$.

31. Show that the eigenvalues of a real symmetric matrix are real.

Answer **any 2** questions from among the questions from **32** to **35**. **Each** question carries **15** marks.

32. Show that the equation $x^4 - 10x^3 + 23x^2 - 6x - 15 = 0$ can be transformed into a reciprocal equation by diminishing its roots by 2. Hence solve the given equation.

33. Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ using Cardan's method.

34. a) Solve $y'' + 16y = e^{-3x} - \cos 4x$.

    b) Solve the initial value problem $x^2y' - y^2y' - 2xy = 0$, $y(1) = 1$.

35. Is the matrix $\begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{bmatrix}$ diagonalizables ? If yes, find the diagonal equivalent.

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